Inverse problems in elastography and displacement-flow MRI

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DOI:
10.33612/diss.112422123

Publication date:
2020

Chapter 5

Conclusion

5.1 Conclusion

In chapter 2 we have presented a robust method for estimating velocities from dual-VENC data by PC-MRI. We presented a theoretical analysis of the phase contrast MRI technique under the approach of least squares functionals and under this comprehension we gave a new idea for a more robust and less noisy estimation of the velocity, involving three measurements. We also presented an empirical analysis by making measurements for a phantom and for volunteers. We reconstructed velocities for different combinations of VENCs, and we propose some convenient combinations for the two VENCs used, such that in practice it would be necessary to choose only one of the VENCs. The reconstruction algorithm is relatively simple and it could be implemented by MRI scanners.

The method proposed has the potential of changing the protocols in PC-MRI, since we change from the need of scanning with $|u_{true}| \leq$ VENC, which is in general not known a priori to hold, to scan with high-VENC satisfying $|u_{true}| \leq 3$VENC, which augment the chance to obtain aliasing-free images at the first try.

In chapter 3 we have presented the extension of ODV in the case of the recovery of harmonic displacements by PC-MRI and we showed a practical analysis for certain types of waveforms. We presented the extension for measurements which can lead us to a discrete Fourier transform in time as well. Therefore we presented useful measurements for obtaining the input of different problems in elastography.

In chapter 4 we studied some hybrid inverse problems in elasticity. We focused on time-independent equations in the displacement field, which is a vector-valued solution. We analyzed ellipticity conditions of the PDE problem augmented with interior data: power density measurements and the internal displacements. Since our information is internal, we obtained better stability estimates than boundary value inverse problems.

The inverse problem of linear elasticity with power density measurements was studied in dimension two with the additional knowledge of the pressure, because there is no ellipticity, according to Definition 4.1, in the case of unknown pressure. We obtained ellipticity for two measurements under certain conditions over the small strain tensors which seem natural to impose, and a trivial kernel if in addition we impose a lower bound for the frequency ω. We showed the convergence of a reconstruction algorithm for the recovery of μ. We also applied the techniques applied for this inverse problem in the study of the recovery of μ if the equation has a nonlinear forcing term, which is a differential operator of order at most one. This work is an extension of [Bal14] to the case of elasticity, which is not present in the literature, and using different techniques in computations.

The inverse problem of Saint-Venant elasticity model with internal measurements was studied for dimension two and three. We showed ellipticity for two measurements under certain conditions over the strain tensors. We also obtained a trivial kernel without imposing any condition over the frequency ω, which is a good result if we place this problem in the context of the low hencs in MRE. We finally proposed an algorithm for the reconstruction of μ based in the obtained stability estimates. This work can be seen as an extension of [AWZ15] to the case of the Saint-Venant model.

5.2 Perspectives

The ODV idea certainly has questions to explore about, for example, the possibility of adaptation to, for example, 4D-flow [Sta+14] and displacement encoding with stimulated echoes (DEnSE) [Ale+99].

The ODV technique can be explored in cases of high variability of the recovered parameter, since it is a non-smoothing technique as the other techniques found in the literature. Finally, it would be interesting to combine ODV with other approaches in inverse problems in imaging, for example compressed sensing [Lus+08], in order to accelerate acquisitions or coupling the recovered parameter with a physical model in order to denoise the recovery.

The power density measurements is an example of nonlinear model of measurements in hybrid inverse problems and it is open to study other nonlinear models, for example with any arbitrary power over the norm of the small strain tensor, analogue to [Bal13]. In addition, reasonable extensions are to obtain analog results in dimension three and to assume that the pressure is unknown.

About the nonlinear models of elasticity, it would be interesting to extend the study of the inverse problem for the Saint Venant model to another models, for example the Neo-Hookean model.

In real applications, the functions involved in the models of hybrid inverse problems could be not as smooth as we assume them. Hence, it would be useful to reduce the regularity in the estimates obtained in this work. In addition, the studied problems are time-independent. A possible future work is to study the time-dependent case, which would increase the number of potential applications to other modalities of elasticity imaging.

Finally, it would be interesting to implement the proposed algorithms in hybrid inverse problems, verifying the convergence in reasonable times. This would be an important step for showing the applicability of this work.